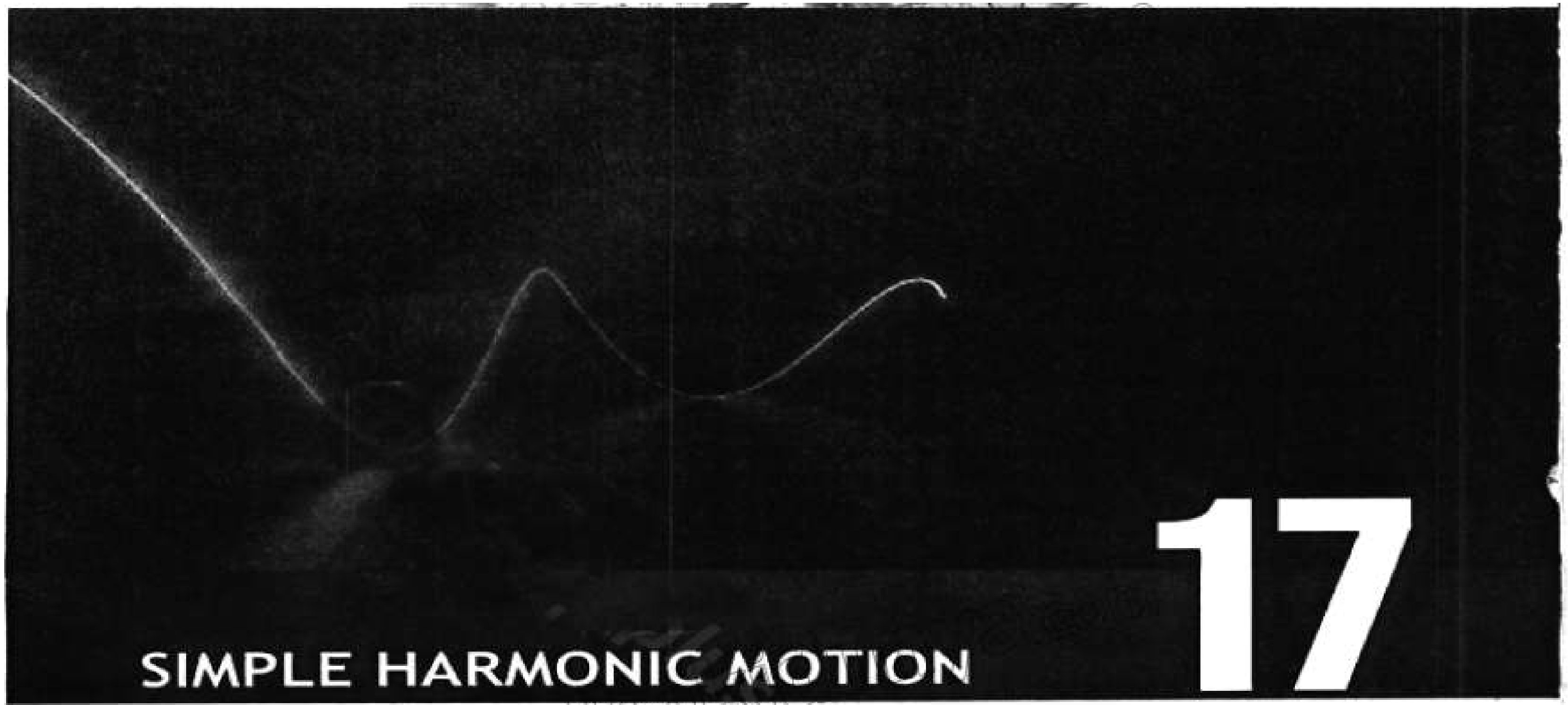


SIMPLE HARMONIC MOTION

17

Student Learning Outcomes (SLOs)

The student will

- describe simple examples of free oscillations.
- use the terms displacement, amplitude, period, frequency, angular frequency and phase difference in the context of oscillations.
- Express the period of simple harmonic motion in terms of both frequency and angular frequency.
- Explain that simple harmonic motion occurs when acceleration is proportional to displacement from a fixed point and in the opposite direction.
- use $a = -\omega^2 x$ to solve problems.
- use the equations $v = v_o \cos(\omega t)$ and $v = \pm\omega\sqrt{x_o^2 - x^2}$ to solve problems.
- Analyze graphical representations of the variations of displacement, velocity and acceleration for simple harmonic motion.
- Analyse the interchange between kinetic and potential energy during simple harmonic motion.
- Apply $\frac{1}{2}m\omega^2 x_o^2$ for the total energy of a system undergoing simple harmonic motion.
- describe that a resistive force acting on an oscillating system causes damping.
- use the terms light, critical and heavy damping.
- sketch displacement-time graphs to illustrate light, critical and heavy damping.
- State that resonance involves a maximum amplitude of oscillations and that this occurs when an oscillating system is forced to oscillate at its natural frequency.
- Describe practical examples of free and forced oscillations.
- Describe practical examples of damped oscillations [with particular reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system.]
- Justify qualitatively the factors which determine the frequency response and sharpness of the resonance.
- identify the use of standing waves and resonance in applications [such as rubens tubes, chladni plates and acoustic levitation (knowledge of wave harmonic modes is not required)]
- Justify the importance of critical damping in a car suspension system.
- Justify that there are some circumstances in which resonance is useful [such as tuning a radio, microwave oven and other circumstances in which resonance should be avoided such as airplane's wing or a suspension bridge].

Have you ever wondered why a pendulum swings back and forth, or how a guitar string vibrates to produce sound? Perhaps you've noticed the smooth motion of a child on a swing or the rhythmic movement of a spring-based toy. These phenomena are all connected by a fundamental concept in physics: Simple Harmonic Motion (SHM). SHM can be observed in various natural and man-made systems. It is in the rhythms of nature, how the universe moves, vibration of atoms, mechanical systems that engineers create, and countless other systems from everyday life.

In this chapter, we'll delve into the world of SHM, exploring topics such as oscillations, uniform circular motion, phase of motion, and energy conservation. We'll also examine the differences between free and forced oscillations, and discover the fascinating phenomenon of resonance.

Sometimes harmonic motion can cause problems. One famous example is the collapse of Tacoma Narrows Bridge. The bridge was designed by structural engineers who did not adequately take into account the role of harmonic motion and it led into its collapse. So, let's go to uncover the rhythm of such a motion!

17.1 OSCILLATIONS

What does a child on a swing, the pendulum of a clock and bouncing of children on the trampoline (as shown in Fig.17.1), all have in common? They all oscillate, i.e., they move back and forth between two points.

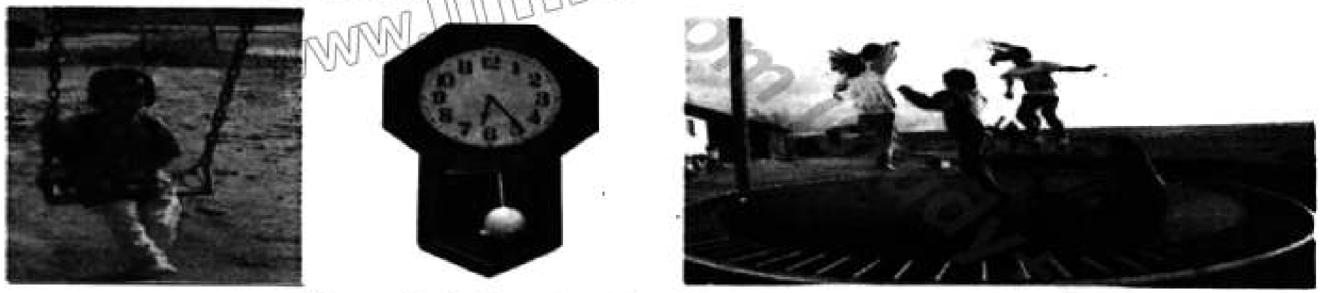


Figure 17.1: Examples of oscillation in our surrounding.

When a body moves to and fro about its mean position, then such motion is called vibratory or oscillatory motion.

In our surrounding, there are many other systems which oscillate. For examples,

- The motion of the Earth during earthquake.
- The wings of birds during flying.
- A string of a guitar producing music.
- Vibration of atoms in a crystal.
- The beating of heart.

For Your Information

Ocean waves or ripples on a pond exhibit oscillatory motion. Sound waves propagate through a medium, causing particles to oscillate. Light waves and other electromagnetic radiation exhibit oscillatory behavior.

- Mass attached to a compressed or stretched spring (horizontally or vertically). Once it is released, starts oscillating.

The complete round trip of an oscillating or vibrating body about its *mean position* is called oscillation or vibration.

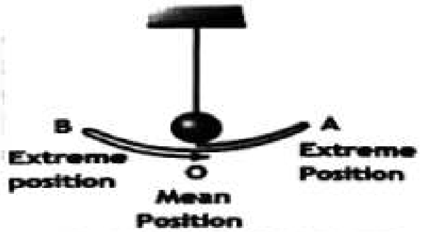


Figure 17.2: One oscillation.

One oscillation occurs when a particle moves from its mean position to extreme position (A) in one direction, moving back to extreme position (B) in the opposite direction through mean position and back once more to mean position, as shown in Fig. 17.2.

Terms Related to Oscillations

In this section, we shall know about some important terms related to oscillatory motion.

(i) Displacement (x): The distance of vibrating body from the equilibrium position on either side at any instant of time is known as displacement. Its SI unit is meter (m).

(ii) Amplitude (x_o or A): The maximum displacement of a vibrating body from mean position is called amplitude. Its SI unit is meter (m).

(iii) Time Period (T): The time taken to complete one oscillation or vibration is called time period of the oscillation. It is denoted by T. Its SI unit is second (s).

(iv) Frequency (f): The number of vibrations or oscillations (n) per unit time (t) is called frequency. It is denoted by f, and $f = \frac{n}{t}$. Its SI unit is hertz (Hz).

One hertz is defined as: the one oscillation per second. The relationship between frequency and time period is:

$$f = \frac{1}{T}$$

(v) Angular Frequency (ω): Angular displacement per unit time is called angular frequency. It is related to the frequency 'f' and time period 'T' of the oscillation by the expressions as:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

17.2 SIMPLE HARMONIC MOTION

Simple Harmonic Motion is a type of motion in which a system oscillates back and forth around a mean (equilibrium) position. It is characterized by a restoring force. Simple harmonic motion (SHM) is defined as:

Such oscillatory motion in which acceleration of a particle is directly proportional to its displacement from the mean position and is always directed towards the mean position.

Mathematically, simple harmonic motion is expressed as:

$$a \propto -x$$

SHM can be observed in many natural phenomena. A good example of SHM is motion of mass (m) attached to an elastic spring. The other end of spring is connected to a fixed support, as shown in Fig. 17.3.

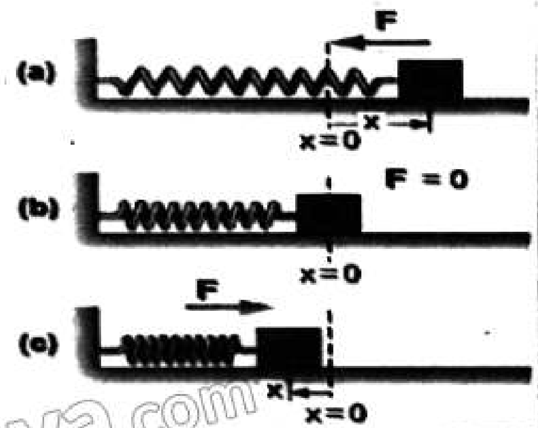


Figure 17.3: Mass 'm' oscillates on horizontal surface.

Let us assume that the mass of spring is ignored and the mass is free to move on a frictionless, horizontal surface. In the absence of an external force, the spring is neither stretched nor compressed. Hence, the mass stays at its equilibrium or mean position, which we identify as x = 0, as shown in Fig. 17.3 (b).

Let the mass is stretched towards right through a displacement 'x' from its equilibrium position by applying a force F, as shown in Fig. 17.3 (a). Due to elasticity, spring exerts an opposite force on the mass which is proportional to the displacement 'x' from mean position. This force is called restoring force. This restoring force obeys **Hooke's law**, i.e.,

$$F = -k\,x \qquad \text{\_\_\_\_\_\_ (17.1)}$$

Here 'k' is called the spring constant. The SI unit of k is N m^{-1}.

The negative sign in Eq. (17.1) shows that the force exerted by spring is always directed opposite to the displacement of the mass from mean position.

When the mass is displaced to the right of mean position, the displacement x is positive but the restoring force is directed to the left (mean position).

When the mass is released, it moves towards the mean position. Due to inertia, it cannot stop at mean position and goes ahead. Then the mass begins to compress the spring and slows down, coming to rest at the left side of the mean position equal to its initial distance on right side, as shown in Fig. 17.3 (c).

The compressed spring then pushes the mass back toward the mean position. Again, the mass cannot stop at the mean position and goes ahead. The result is that the mass oscillates back and forth about the mean position. Since the mass is continuously changing its direction, so it accelerates. According to **Newton's second law of motion**, the force acting on the mass is given by:

$$F = m\,a \qquad \text{\_\_\_\_\_\_ (17.2)}$$

By comparing Eq. (17.1) and Eq. (17.2), we get:

$$m a = -k x$$

or

$$a = -\frac{k}{m} x \quad \text{(17.3)}$$

In Eq. (17.3), $\frac{k}{m} = \omega^2$ is constant. So, angular frequency of 'mass attached to the spring' is:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(17.4)}$$

Now Eq. (17.3) can be written as:

$$a \propto -x$$

This relation shows that the oscillating mass exhibits SHM; its acceleration (a) is proportional to its displacement (x) and is directed towards mean position.

For Your Information

The atoms in a material oscillates as if the atoms were connected by a tiny spring, hence exhibit SHM. The spring constant of this spring depends upon the type of bonding between the atoms. This model can be extended to solids, where atoms are often thought of as being connected to their neighbours by springs. This leads to an experimental way of obtaining information about interatomic forces in the solids.

Frequency and Time Period of an Oscillating Mass-Spring System

As angular frequency 'ω' is related to the frequency and time period by the following expressions:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

So,

$$T = \frac{2\pi}{\omega}$$

By putting $\omega = \sqrt{\frac{k}{m}}$, we get:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Similarly, for the frequency, we can get the following expression:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Example 17.1: A Fish is hung on a spring scale.
(a) What is the constant of the spring in such a scale if the spring stretches 8.0 cm for a 10.0 kg load? (b) What is the mass of the fish that stretches the spring 5.5 cm?

(a) Given: $x = 8.0$ cm $= 0.08$ m $\quad m = 10.0$ kg
To Find: $k = ?$

Solution: According to the Hooke's law, $F = kx$
Here, $F = W = mg$.

Therefore, $$k = \frac{mg}{x}$$

Putting values, we get: $$k = \frac{10 \times 9.8}{0.08} = 1225 \text{ N m}^{-1}$$

(b) Given: $x = 5.5$ cm $= 0.055$ m
To Find: $m = ?$

Solution: As $$m = \frac{kx}{g}$$

Putting values, we get: $$m = \frac{1225 \times 0.055}{9.8} = 6.875 \text{ kg}$$

Assignment 17.1

An object with mass 500 g is suspended from a spring. The spring is stretched by 9.8 cm. Calculate the spring constant.

17.3 UNIFORM CIRCULAR MOTION AND SHM

There is a close connection between circular motion and simple harmonic motion. Therefore, many aspects, such as displacement, velocity and acceleration of simple harmonic motion (SHM) can be understood by relating it with uniform circular motion.

Consider a point P is moving on a circular path of radius x_o in xy-plane at constant angular velocity ω, as shown in Fig. 17.4 (a). At the same time, Q be the projection of P on the x-axis undergoes oscillation along the diameter of circular path between $-x_o$ and $+x_o$.

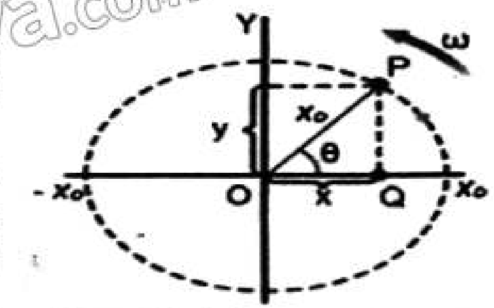


Figure 17.4 (a): A point P is moving on a circular path at constant angular velocity ω.

It is seen that the time period of one revolution of point P on the circular path is equal to the time period of one oscillation of point Q on the diameter. Therefore, the angular speed of P is the same as the linear speed of Q. Thus, the expressions for displacement, velocity and acceleration of P also hold for the Q.

Expressions for Displacement

As the point P moves on the circle, at some instant t, the angle made by the line OP with the x-axis is $\theta = \omega t$. From the Fig. 17.4 (a), the instantaneous displacement x of Q at that instant can be calculated as:

From ΔOPQ, $$\cos\theta = \frac{x}{x_o}$$

or $\qquad x = x_o \cos\theta = x_o \cos\omega t \qquad$ ______ (17.5)

Eq. (17.5) can be used for calculating displacement of a body executing simple harmonic motion.

Expressions for Velocity

Velocity of the point P at instant t is $v_P = x_o\,\omega$, which is directed along the tangent. Instantaneous velocity v of the point Q is the projection of v_P on the x-axis, which is the horizontal component of v_P, as shown in Fig. 17.4 (b).

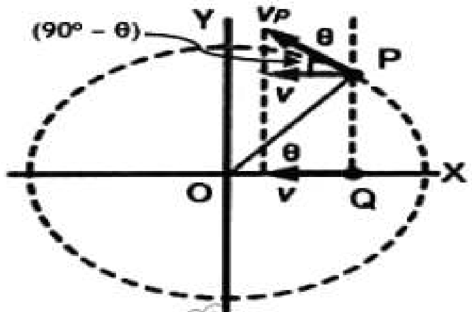


Figure 17.4 (b): Velocity of the point P and its horizontal component.

$$v = v_P \cos(90^\circ - \theta)$$

or $\qquad v = v_P \sin\theta \qquad$ ______ (17.6)

as, $\sin^2\theta + \cos^2\theta = 1$ or $\quad \sin^2\theta = 1 - \cos^2\theta$

so, Eq. (17.6) becomes:

$$v = v_P \sqrt{1-\cos^2\theta}$$

or $\qquad v = x_o\,\omega \sqrt{1-\cos^2\theta}$

Using Eq. (17.5), we get:

$$v = \omega \sqrt{x_o^2 - x^2} \qquad \text{______ (17.7)}$$

Eq. (17.7) can be used for calculating velocity of a body executing simple harmonic motion.

Expressions for Acceleration

Since ω is constant, so the acceleration of the point P is centripetal i.e., $a_P = x_o\,\omega^2$, which is directed inward towards O. The acceleration 'a' of Q is the horizontal component of a_P, as, shown in Fig. 7.4 (c).

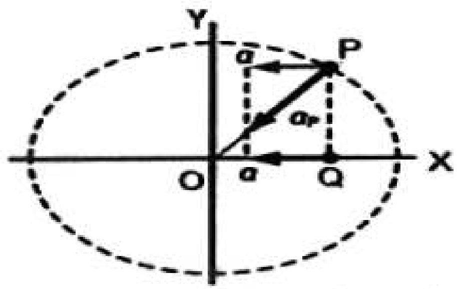


Figure 17.4 (c): Acceleration of the point P and its horizontal component.

$$a = -\,a_P \cos\theta = -\,x_o\,\omega^2 \cos\theta$$

Since the velocity is decreasing, so the negative sign shows that acceleration of Q is directed towards O. Using $\cos\theta = \dfrac{x}{x_o}$, we get:

$$a = -\,\omega^2 x \qquad \text{______ (17.8)}$$

This equation describes the SHM. Hence, the oscillatory motion of point Q is SHM; its acceleration is proportional to its displacement and is directed opposite to the displacement from mean position. Hence, we can conclude that:

When a body moves in a circle, its projection undergoes simple harmonic motion on the diameter of the circle.

So, acceleration of a body executing simple harmonic motion can be calculated by using Eq. (17.8).

Example 17.2: A body with mass 800 g attached to a spring, vibrates with amplitude 30 cm. The restoring force is 60 N when the displacement is 0.30 m. (a) Find out its angular frequency (b) Also calculate magnitude of its velocity and acceleration at x = 12 cm.

Given: m = 800 g = 0.8 kg $\quad$ x_o = 30 cm = 0.30 m

F = 60 N at x_o = 30 cm = 0.30 m $\quad$ x = 12 cm = 0.12 m

To Find: (a) angular frequency = ω = ?

(b) velocity = ? $\quad$ Acceleration = a = ?

Solution: (a) As
$$k = \frac{F}{x_o} = \frac{60}{0.30} = 200 \text{ N m}^{-1}$$

Angular frequency is given by the relation:
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.8}} = 15.81 \text{ rad s}^{-1}$$

(b) For velocity, we use:
$$v = \omega\sqrt{x_o^2 - x^2}$$

Putting values, we get:
$$v = 15.81\sqrt{0.3^2 - 0.12^2} = 1.4 \text{ m s}^{-1}$$

For acceleration, we use:
$$a = \omega^2 x$$

Putting values, we get:
$$a = (15.81)^2 (0.12) = 3 \text{ m s}^{-2}$$

Assignment 17.2

Time period of a mass attached at the end of a spring is 0.40 s. Find out the magnitude of acceleration when the displacement is 4 cm.

17.4 PHASE

We have studied that the instantaneous displacement of the point executing SHM is given by the Eq. (17.5), as:

$$x = x_o \cos\theta \qquad \text{or} \qquad x = x_o \cos\omega t$$

Here, $\theta = \omega t$ is the phase of motion and can be defined as:

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of a point executing SHM is called phase of the motion.

In general, the Eq. (17.5) can be written as:

$$x = x_o \cos(\omega t + \varphi) \qquad (17.9)$$

The term φ is called initial phase. The inclusion of φ gives the information regarding the starting or initial phase of oscillation.

Physical Significance of Phase

To give the physical significance of phase, we use displacement-time graph. Since x is periodic, so T is the time period of the oscillation.

If $\varphi = 0°$, then Eq. (17.9) becomes:

$$x = x_o \cos\omega t$$

Putting t = 0, T/4, T/2, 3T/4, T ...., we get a graph, as shown in Fig. 17.5 (a). This graph shows that:

At t = 0, T/2 and T (corresponding to $\theta = 0$, π and 2π), the point is at the extreme positions.

At t = T/4 and 3T/4 (corresponding to $\theta = \pi/2$ and $3\pi/2$), the point is at mean position.

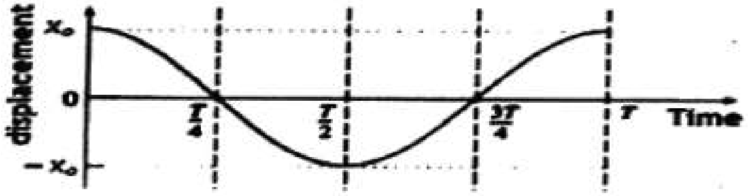


Figure 17.5 (a): Graph of $x = x_o \cos(\omega t + \varphi)$, for $\varphi = 0°$.

If $\varphi = 90°$, then Eq. (17.9) becomes:

$$x = x_o \cos(\omega t + 90°)$$

Putting t = 0, T/4, T/2, 3T/4, T ...., we get a graph, as shown in Fig. 17.5 (b). This graph shows that:

At t = 0, T/2 and T (corresponding to $\theta = 0$, π and 2π), the point is at mean positions.

At t = T/4 and 3T/4 (corresponding to $\theta = \pi/2$ and $3\pi/2$), the point is at extreme positions.

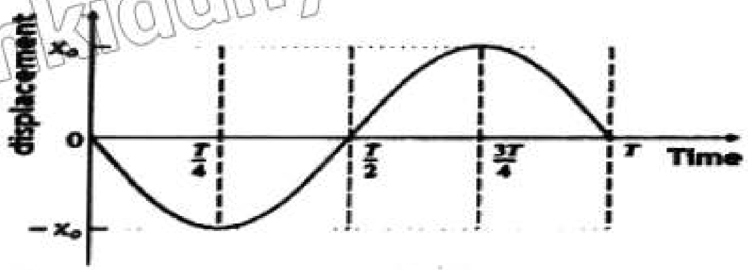


Figure 17.5 (b): Graph of $x = x_o \cos(\omega t + \varphi)$, for $\varphi = 90°$.

If $\varphi = 180°$, then Eq. (17.9) becomes:

$$x = x_o \cos(\omega t + 180°)$$

Putting t = 0, T/4, T/2, 3T/4, T ...., we get a graph, as shown in Fig. 17.5 (c).

It can be noted that:

- The curve in Fig. 17.5 (b) leads the curve in Fig. 17.5 (a) by 90°, because their phase difference is 90°.

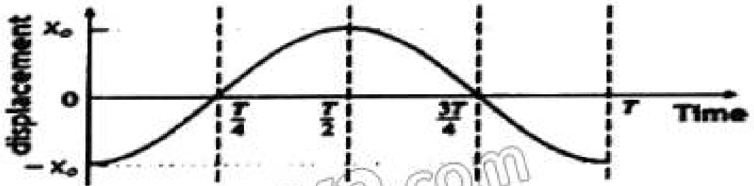


Figure 17.5 (c): Graph of $x = x_o \cos(\omega t + \varphi)$, for $\varphi = 180°$.

- Similarly, the curve in Fig. 17.5 (c) leads the curve in Fig. 17.5 (a) by 180°, because their phase difference is 180°.

When the phase difference between two oscillating systems is 180°, they are said to be oscillating out of phase.

When the phase difference between two oscillating systems is 0° or 360°, they are said to be oscillating in phase.

17.5 GRAPHICAL REPRESENTATIONS OF DISPLACEMENT, VELOCITY AND ACCELERATION FOR SHM

The graphical representations of displacement, velocity and acceleration of a body executing SHM is given in Fig. 17.5 (d).

(i) The displacement of the particle executing SHM is given by the expression:

$$x = x_o \cos\omega t \quad \text{\_\_\_\_\_\_ (i)}$$

As maximum value of displacement of the particle is x_o.

(ii) The velocity of the particle executing SHM is given by the expression:

$$v = x_o\, \omega \sqrt{1-\cos^2\theta}$$

or $$v = x_o\, \omega \sin(\omega t) \quad \text{\_\_\_\_\_\_ (ii)}$$

The velocity of the particle is maximum (i.e., $v = \pm x_o\, \omega$) at the mean position and zero at the extreme positions.

(iii) The acceleration of the particle executing SHM is given by the expression:

$$a = -\omega^2 x$$

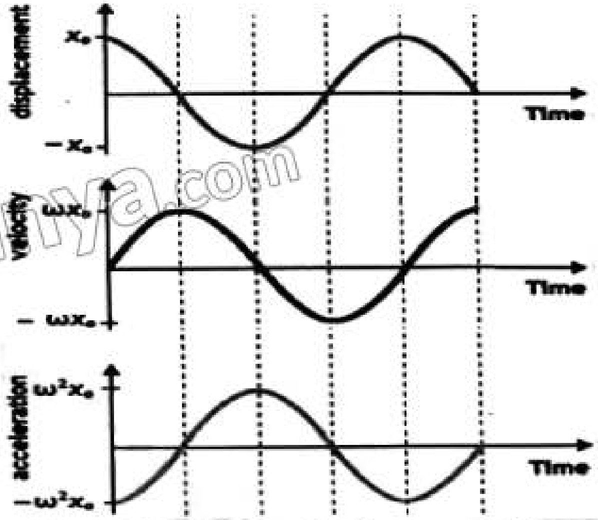


Figure 17.5 (d): Graphical representations of 'x', 'v' and 'a' of a body executing SHM.

Putting $x = x_o \cos\omega t$, we get:

$$a = -x_o\, \omega^2 \cos(\omega t) \quad \text{\_\_\_\_\_\_ (iii)}$$

The acceleration will be maximum (i.e., $x_o\, \omega^2$) at the extreme positions, and zero at the mean position.

The graph of the Eqs. (i), (ii) and (iii) is shown in Fig. 17.5 (c). It can be seen that:

- The phase difference between velocity and displacement is $\pi/2$.
- The phase difference between acceleration and displacement is π.

17.6 SIMPLE PENDULUM

An ideal simple pendulum consists of a small but heavy bob of mass m which is suspended by a light and inextensible string of length *l*. The other end of the string is attached to a fixed frictionless support at point P, as *shown in Fig.* 17.6 (a).

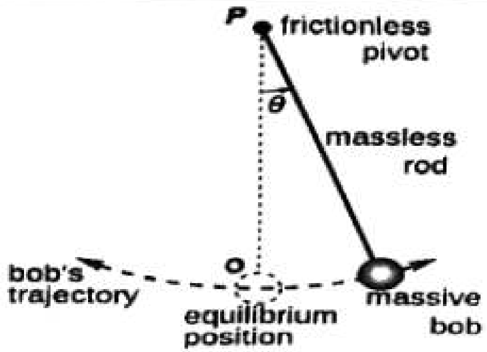


Figure 17.6 (a): Simple pendulum.

When the bob is displaced slightly from its mean position O, after releasing it oscillates to-and-fro along the arc of a circle with centre at P. Suppose that the oscillating bob is at point A at some instant, where the displacement is x and the angle OPA = θ. The forces acting on the bob are the tension T exerted by the string and its weight mg acting vertically downward.

The weight mg of the bob can be resolved into its two rectangular components, i.e.

i) $mg\cos\theta$ along the string.

ii) $mg\sin\theta$ along the tangent to the arc.

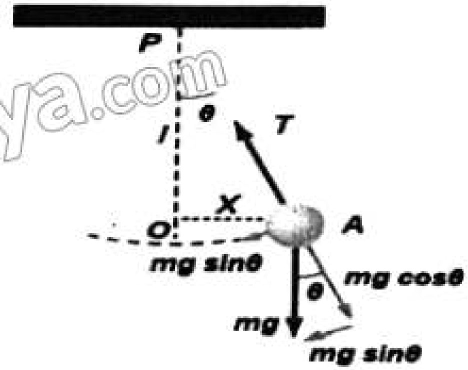


Figure 17.6 (b): Components of weight.

Tension in the string exactly cancels the component $mg\cos\theta$. The net force $mg\sin\theta$ on the bob at A is the restoring force which makes it to accelerate towards equilibrium position, i.e.,

$$F = -mg\sin\theta \quad \text{\_\_\_\_\_\_ (i)}$$

Negative sign indicates that the force is acting towards mean position O. When θ is small (less than 10° or 0.2 rad), as shown in Table (7.1), we make the approximation sin θ = θ and Eq. (i) becomes:

$$F = -mg\theta \quad \text{\_\_\_\_\_\_ (ii)}$$

From Fig. 7.6 (b), arc length = OA, then,

$$\theta = \frac{OA}{l}$$

For small angle θ, the arc OA = x, then Eq. (iii) becomes,

$$F = -mg\frac{x}{l} \quad \text{\_\_\_\_\_\_ (iii)}$$

According to Newton's second law of motion, the force acting on the bob is given by:

$$F = ma \quad \text{\_\_\_\_\_\_ (iv)}$$

Table 7.1: Values of θ and sin θ for small angle.

θ (degree)	θ (radian)	sin θ
0.00	0.0000	0.0000
1.00	0.0175	0.0175
2.00	0.0349	0.0349
3.00	0.0524	0.0524
4.00	0.0698	0.0698
5.00	0.0873	0.0872
6.00	0.1048	0.1046
7.00	0.1222	0.1219
8.00	0.1397	0.1392
9.00	0.1571	0.1565
10.00	0.1746	0.1737

By comparing Eq. (iii) and Eq. (iv), we get:

$$m\,a = -m\,g\,\frac{x}{l}$$

$$a = -\frac{g}{l}\,x \qquad \text{\_\_\_\_\_\_ (v)}$$

or $$a \propto -x \qquad \text{\_\_\_\_\_\_ (vi)}$$

The Eq. (vi) describes the SHM. Hence, an oscillating simple pendulum exhibits SHM. As equation for acceleration of a body executing SHM is:

$$a = -\omega^2 x \qquad \text{\_\_\_\_\_\_ (vii)}$$

By comparing Eq. (v) and Eq. (vii), we get:

$$\omega = \sqrt{\frac{g}{l}} \qquad \text{\_\_\_\_\_\_ (viii)}$$

So, the expression for time period of the simple pendulum can be obtained by putting Eq. (viii) in $T = 2\pi/\omega$, thus we get:

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{\_\_\_\_\_\_ (17.10)}$$

Above equation shows that:

The time period of a simple pendulum depends on the length of the pendulum and the acceleration due to gravity. It is independent of the mass.

Example 17.3: A simple pendulum completes one vibration in one second. Calculate its length when $g = 9.8\ m\ s^{-2}$.

Given: Time period of the simple pendulum = $T = 1\ s$

Gravitational acceleration = $g = 9.8\ m\ s^{-2}$

To Find: Length of the simple pendulum = $l = ?$

Solution: As $$T = 2\pi\sqrt{\frac{l}{g}}$$

Putting values, we get: $$1 = 2 \times 3.14\sqrt{\frac{l}{9.8}}$$

$$l = 0.248\ m = 24.8\ cm$$

Assignment 17.3

Find the time periods of a simple pendulum with length 1 m, placed on Earth and on Moon. The value of g on the surface of Moon is 1/6th of its value on Earth.

17.7 ENERGY CONSERVATION IN SHM

Energy of a body executing SHM remains conserved. To examine this fact, we again consider a vibrating mass-spring system, as shown in Fig. 17.7 (a). When the spring is stretched by the applied force 'F' through a displacement 'x', work must be done. This work W is given by:

$$W = F_{av}\ x \quad \text{\_\_\_\_\_\_ (i)}$$

According to Hooke's law, the applied force is given by:

$$F = k\,x \quad \text{\_\_\_\_\_\_ (ii)}$$

Since the force increases linearly from 0 to kx, the average force F_{av} is given by:

$$F_{av} = \frac{0 + k\,x}{2} = \frac{1}{2}k\,x \quad \text{\_\_\_\_\_\_ (iii)}$$

Substituting the value of F_{av} in Eq. (i) from Eq. (iii), we get:

$$W = \left(\frac{1}{2}k\,x\right)(x) = \frac{1}{2}k\,x^2$$

As, this work is stored in spring as elastic potential energy. Hence, the potential energy at any instant 'x' is given by:

$$P.E = \frac{1}{2}k\,x^2 \quad \text{\_\_\_\_\_\_ (17.11)}$$

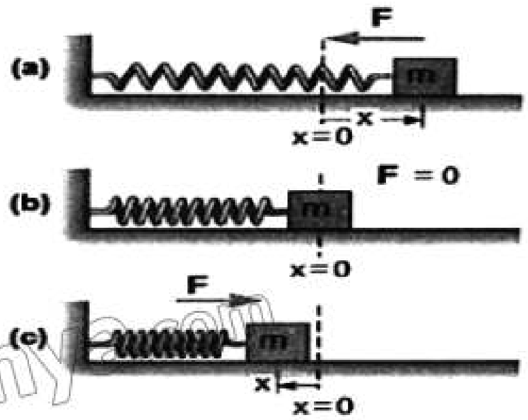


Figure 17.7: A vibrating mass-spring system.

At extreme position, where the displacement is maximum, i.e. $x = x_o$, the block is at rest. So, its kinetic energy is zero and total energy is entirely elastic potential energy, i.e.

$$T.E = P.E_{max} = \frac{1}{2}k\,x_o^2 \quad \text{\_\_\_\_\_\_ (17.12)}$$

When the block is released, it moves toward mean position. As a result, its velocity increases and also its kinetic energy. However, the displacement of the block decreases and also its elastic potential energy.

As, the instantaneous velocity v of the block executing SHM is given by,

$$v = \omega\sqrt{x_o^2 - x^2} \quad \text{\_\_\_\_\_\_ (iv)}$$

As, $\omega = \sqrt{\frac{k}{m}}$, so Eq. (iv) becomes:

$$v = \sqrt{\frac{k}{m}(x_o^2 - x^2)} \quad \text{\_\_\_\_\_\_ (v)}$$

At mean position x = 0, the block gets maximum velocity, i.e.,

$$v_o = \sqrt{\frac{k}{m}}\ x_o \quad \text{\_\_\_\_\_\_ (vi)}$$

Hence, at any instant where the displacement is 'x', the kinetic energy is given by:

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}k(x_o^2 - x^2) \quad \text{\_\_\_\_\_\_ (17.13)}$$

At mean position ($x = 0$), the elastic potential energy is zero but the velocity is maximum, hence the total energy is entirely kinetic energy.

$$T.E = K.E_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}k\,x_o^2 \quad \text{———} \quad (17.14)$$

Let us now examine the total energy of the systems at displacement x. As, the total energy is the sum of kinetic and potential energy, i.e.

$$T.E = K.E + P.E \quad \text{———} \quad (vii)$$

After substituting the values of K.E and P.E from Eq. (17.11) and Eq. (17.13) in Eq. (vii), we get:

$$T.E = \frac{1}{2}k(x_o^2 - x^2) + \frac{1}{2}k\,x^2 = \frac{1}{2}k\,x_o^2 \quad \text{———} \quad (17.15)$$

As, $\omega = \sqrt{\frac{k}{m}}$, or $k = m\,\omega^2$, so Eq. (17.15) can also be written as:

$$T.E = \frac{1}{2}m\omega^2 x_o^2 \quad \text{———} \quad (17.16)$$

From Eqs. (17.12), (17.14) and (17.15), it is proved that:

Total energy of the mass-spring system is constant and is proportional to square of the amplitude of oscillation.

This statement of conservation of energy is equally valid for all bodies executing SHM.

The energy oscillates between K.E and P.E, but their sum remains constant. This can be illustrated by plotting the graph of K.E, P.E and T.E verses displacement, as shown in Fig. 17.7 (d).

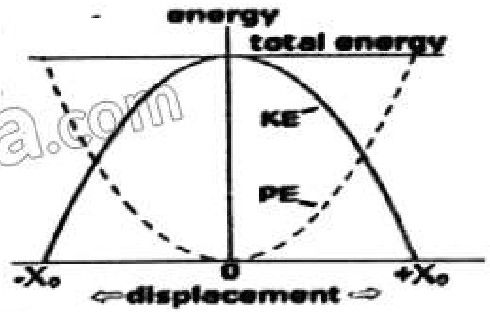


Figure 17.7 (d): Graph of energy (K.E, P.E and T.E) against displacement, for SHM.

Example 17.4: A 0.025 kg mass is attached to a spring which is displaced through 0.10 m to right of its mean position and then released. Time period of its oscillation is 1.57 s. Calculate its:

(a) Angular speed (b) The total energy (c) The maximum acceleration.

Given: $m = 0.025$ kg $\quad x_o = 0.10$ m $\quad T = 1.57$ s

To Find: (a) Angular Speed = ω =? (b)Total Energy = $T.E$ = ?
(c) Maximum Acceleration = a = ?

Solution: (a) To find angular speed, we use the relation:

$$\omega = \frac{2\pi}{T}$$

Putting values, we get: $\omega = \frac{2\,(3.14)}{1.57} = 4\text{ rad s}^{-1}$

(b) The total energy can be found by using the relation:

$$T.E = \frac{1}{2} m \omega^2 x_o^2$$

Putting values, we get: $T.E = \frac{1}{2}(0.025)(4)^2(0.1)^2 = 2 \times 10^{-3}\ J$

(c) For the maximum acceleration, we use the relation:

$$a = x_o \omega^2$$

Putting values, we get: $a = 0.1 \times 4^2 = 1.6\ m\ s^{-2}$

Assignment 17.4

Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant t is given by $x = 0.25 \cos(\pi/8)t$. Also find the displacement of the object after 2.0 s

17.8 FREE, FORCED AND DAMPED OSCILLATIONS

Depending upon the situation, oscillations may be damped, free and forced oscillations. Here we discuss these three types of oscillations in detail.

Free Oscillations

Every oscillator has a natural frequency of vibration with which it vibrates freely after an initial disturbance. For example, an ideal simple pendulum oscillates freely with its natural frequency, when slightly displaced from its mean position, as shown in Fig.17.8 (a). The natural frequency of pendulum depends on its length. If you change the length of string, you may change its natural frequency.

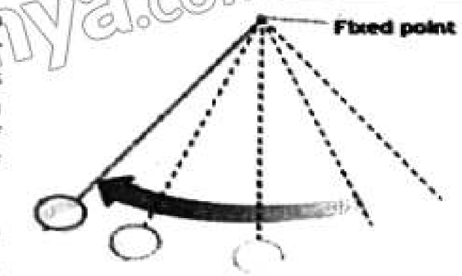


Figure 17.8 (a): Illustration of free oscillation.

If you pluck a guitar string, it continues to vibrate for some time after you have released it. It vibrates with its natural frequency and it gives rise to the particular note that you hear. The natural frequency of guitar string depends on its length. If you change the length of string, certainly you change its natural frequency.

A body is said to be executing free oscillations when it oscillates under the influence of a restoring force without any external force acting on it.

The free oscillations possess constant amplitude and period without any external force acts on it. Ideally, free oscillations do not undergo damping.

Forced Oscillations

If an external force acts on an oscillator, it can change its amplitude of oscillations. The external force shifts the energy to the oscillator at a certain frequency, not necessarily the same as the natural frequency of the oscillator. This frequency is called driven frequency.

For example, when you push a swing, you have to keep periodically pushing it so that the it doesn't reduce its amplitude and continue oscillating. These oscillations are called forced oscillations.

If the external forces make the object to oscillate at the frequency of applied force rather than its natural frequency, then such oscillations are called forced oscillations.

Movement of the pendulum of a clock (as shown in Fig.17.8-b) is also an example of a forced oscillations, because it is driven by a small motor.

Figure 17.8 (b): Illustration of force oscillation.

Damped Oscillations

Damping is the effect of resistive forces which dissipate energy from a vibrating object. When a simple pendulum is being set into oscillatory motion, then after some time, it stops oscillating due to air resistance. All oscillating systems experience such type of resistive force which are known as damping forces. Due to the damping force, amplitude of oscillation decreases over time from one oscillation to the next and eventually stops oscillating.

Examples of damping forces can include frictional forces between moving parts, air resistance or internal forces such as those in springs that tend to dissipate energy as heat.

We know that in reality, a spring won't oscillate forever. Frictional forces will diminish the amplitude of oscillations until eventually the system comes to rest.

If the amplitude of oscillations decreases under damping forces, then such oscillations are called damped oscillations.

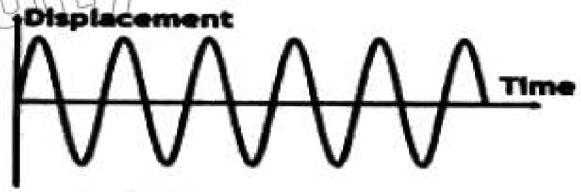


Figure 17.9 (a): Displacement-time graphs to illustrate undamped oscillations.

It can also be defined as: when an oscillator undergoes oscillations before coming to rest under the action of damping force, then such oscillations are called damped oscillations. As an oscillator vibrates, it performs work against force of friction, which result in the gradually decrease of oscillator's energy.

The damping is said to be light when the amplitude of oscillations decreases gradually with time.

Light damping gradually reduces the energy and amplitude of the vibrating object. An example of 'light damping' is a swing in playground, which gradually comes to rest when oscillates freely. Displacement-time graph of light damping is shown in Fig. 17.9 (a) by black curve.

Heavy damping takes long time before the object comes to rest; it is shown in Fig. 17.9 (b) by green curve.

In heavy damping, the oscillator merely moves back to its equilibrium position gradually without completing a single oscillation. It is the result of a very large resistive force.

Oscillations of a mass attached to a spring that is placed in a thick, viscous liquid, can be considered as heavy damping.

Critical damping returns the object to the equilibrium position in the shortest time possible. An example of critical damping is shock absorbers in a car; they increase the resistive force so that after being displaced when going over a bump, the vehicle returns to its original position without oscillating. Critical damping is shown in Fig. 17.9 (b) by red curve.

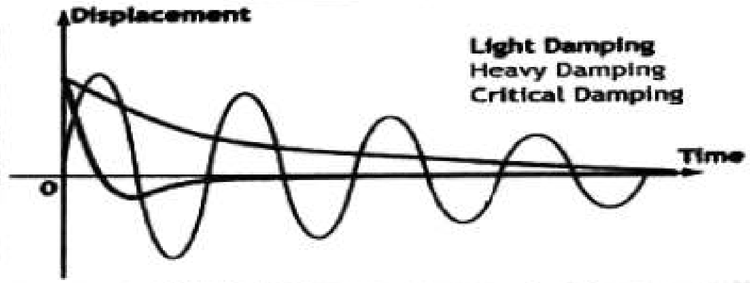


Figure 17.9 (b): Displacement-time graphs to illustrate light, critical and heavy damping.

When an oscillator comes to rest without any oscillation in the shortest time under damping force, then such damping is called critical damping.

Critical damping is often useful in an oscillating system because such systems return to equilibrium position rapidly after facing external deriving force. For example, when a car is passing over a bump on road, it would move up and down violently for sometimes which may cause injury to passengers. To overcome this problem, energy-absorbing devices (damping) known as shock absorbers are positioned parallel to the spring in automobiles. Vehicles have springs between the wheels and the frame, as shown in Fig. 7.9 (c), provide a smoother and comfortable ride.

Figure 17.9 (c): Shock absorbers are fluid-filled tubes that turn the simple harmonic motion of the springs into damped harmonic motion.

Modern auto suspensions are set up so that all of a spring's energy is absorbed by the shock absorbers, eliminating vibrations in single oscillation. This prevents the car from continually bouncing.

17.9 RESONANCE

A driving force is always required to initiate the oscillations of any oscillator, damped or un-damped, by supplying the initial energy for the motion. The driving forces have their own frequencies, which cause the oscillator to vibrate at the driving frequency rather than at the natural frequency.

When the frequency of driving force gradually increases from zero, the oscillator begins to vibrate with small amplitude. As we increase the frequency, the amplitude of vibration also increases. The closer the deriving frequency to the natural frequency, the more efficiently the driving force transfers energy to the oscillator and the greater the resulting amplitude. When the deriving frequency equals the natural frequency of oscillator, the amplitude of vibrations reaches to its maximum value, this situation is called resonance. Hence resonance is defined as:

Resonance is the phenomena in which the amplitude of vibration of an oscillator attains maximum value when deriving frequency becomes equal to the natural frequency of oscillator. The natural frequency of the oscillator is called resonance frequency.

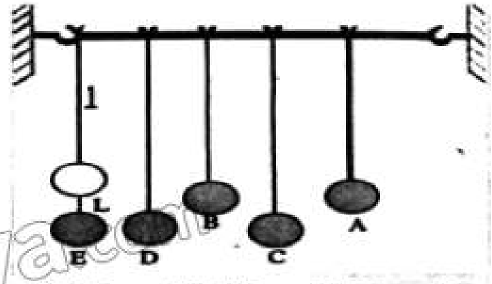


Figure 17.10: Setup to demonstrate resonance.

At resonance frequency, the efficiency of energy transfer from the driving force to the oscillator is maximum. This can be demonstrated with a simple experiment, as shown in Fig. 17.10. Here the five pendulums A, B, C, D and E are suspended vertically from the same horizontal rod. The lengths of A and B are same and equal to l, and the length of C and D are same and equal to L.

If length of pendulum E is made equal to the lengths of pendulum A and B, and E is set into vibrations in direction perpendicular to the plane of the paper. Then, after some time A and B start vibrations automatically but C and D will remain at rest. This is due to the reason that E has same length and frequency as A and B.

If length of pendulum E is made equal to the lengths of pendulum C and D, and pendulum E is set into vibrations in a direction perpendicular to the plane of the paper. Then after some time pendulum C and D start vibrating automatically but pendulum A and B will remain at rest. This is due to the reason that pendulum E has same length and hence frequency as pendulum C and D.

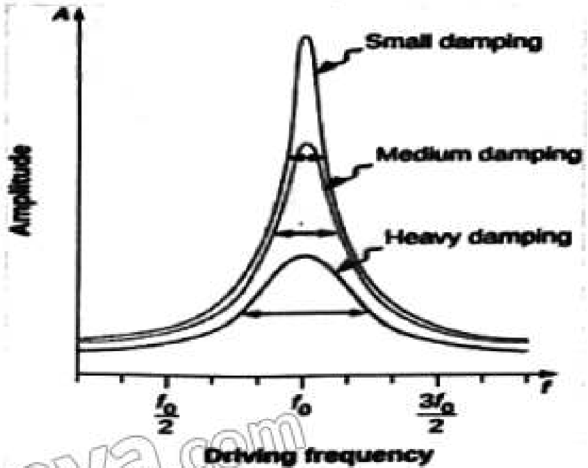


Figure 17.11: Graph of amplitude against frequency of driving force for small, medium and heavy damping.

Effect of Damping on Resonance

Damping reduces the maximum amplitude of an oscillator at its resonance frequency and broadens the resonance curve. This can be illustrated by plotting a graph of the amplitude of a damped harmonic oscillator against the frequency of driving force, as shown in Fig. 17.11. The graph shows that smaller damping results in a sharper resonance peak and larger amplitude. While heavy damping, decreases the amplitude.

In practical applications, such as car's suspension system, heavy damping is employed to minimize oscillations and ensure a smooth ride when traveling over bumps and jumps.

Applications of Resonance in Daily Life

Resonance plays very important role in various phenomena of daily life. Here we shall discuss some circumstances in which resonance is useful:

- **Tuning of Radio and Electrical Resonance:** When we turn the knob of a radio to tune a station. We are actually changing the natural frequency of the electrical circuit of receiver to make it equal to the transmission frequency of the radio station. When the two frequencies match with each other, then resonance occurs. In this way, energy absorption from the station is maximum and this is the only station we hear.
- **Magnetic Resonance Imaging:** Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to oscillate by incoming strong radio waves (on the order of 100 MHz). When resonance occurs, maximum energy is absorbed by the nuclei. The pattern of energy absorbed can be used to produce computer enhanced photography.
- **Heating/Cooking of Food in Microwave Ovens and Resonance:** In a microwave oven, the microwave with a frequency similar to the natural frequency of vibration of water or fat molecules are used. When the food is placed in the oven, the water molecules in the food oscillate by absorbing maximum energy from the microwaves. Hence, it causes the food to heat up for cooking. The plastic or glass containers do not heat up in ovens, since they do not contain water molecules.
- **Resonance in Guitar:** When the guitarist strikes the guitar strings, a vibration is produced. The vibration transmits to the hollow wooden box. Thus, creating resonance, and the sound gets amplified.

Following are some circumstances in which resonance should be avoided:

- **Resonance in Bridge:** Soldiers while marching on the bridge are ordered to break their steps. This is because that the vibrations created by the rhythmic march on the bridge cause the bridge to oscillate with its natural frequency. Thus, amplitude of vibrations increases and resonance occurs that causes the bridge to collapse. One of the most studied examples of this is the collapse of Tacoma Narrows Bridge, as shown in Fig. 17.12. The strong continuous wind drove oscillations of the bridge deck that increased in amplitude until it broke apart.

Figure 17.12: Collapse of Tacoma Narrows Bridge in 1940 during a windstorm.

- **Resonance in Airplane's Wing:** The wing is a very flexible part of the airplane. If the periodic vibrations of the wind gust have a frequency equal to the natural structural frequency of the wing, resonance occurs. At resonance, the amplified vibrations of the wing become too large. It eventually leads to its destruction. To avoid resonance, it is important to design the wing such that the natural frequency of the wing does not match the external frequencies of vibrations.

Science Tidbit

A singer can shatter a glass by loudly singing a note that matches the natural frequency of the glass. When the sound gets too loud for the glass to vibrate with large enough amplitude, it shatters the glass.

Applications of Resonance and Standing Waves

There are many applications of resonance in different devices that generate and use standing waves. Some of them are discussed here:

Rubens Tubes: A Rubens tube (also known as a standing wave flame tube) is used to demonstrate acoustic standing waves. It consists of a metal pipe with holes drilled along the top and sealed at both ends. One sealed end is attached to a small speaker or frequency generator while the other end is connecter to a supply of a flammable gas, as shown in Fig. 17.13. The pipe is filled with the gas, and the gas leaking from the perforations produces a resonant flame.

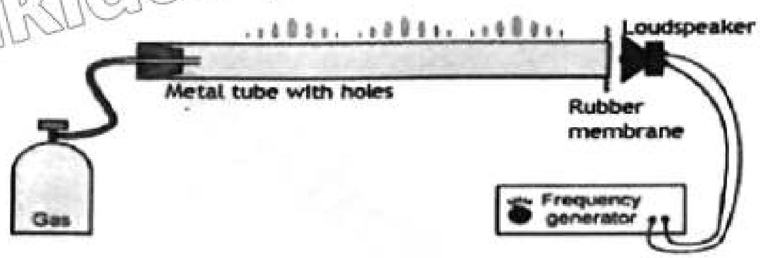
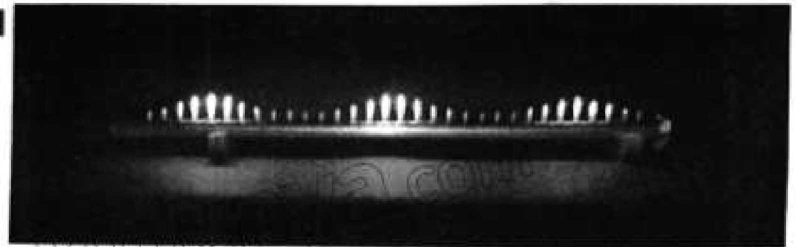


Figure 17.13: Rubens tube setup.

Science Tidbit

Rubens tube is invented by German Physicist Heinrich Rubens in 1905. It graphically shows the relationship between sound waves and sound pressure, acting as a primitive oscilloscope.



When sound waves of resonance frequency (based on the tube dimensions) is produced by frequency generator, a standing wave formed inside the tube. The standing wave creates points with oscillating (higher and lower) pressure within the tube. Less gas will escape from the perforations in the tube where pressure is low, hence the flames will be lower at these points. Large quantity of gas will escape from the perforations in the tube where pressure is high, hence the flames will be high at these points. The wavelength of the standing wave can be determined with a ruler by measuring the distance between low and high flame.

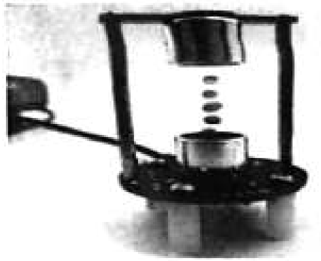
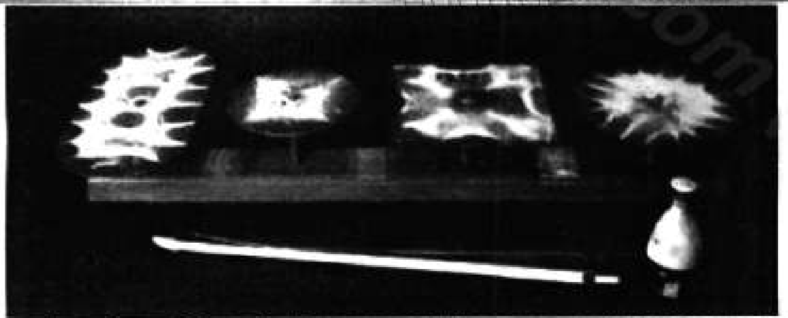


Figure 17.14: Acoustic levitation.

Acoustic Levitation: Acoustic levitation is a method for suspending matter in air against gravity by using acoustic pressure from high intensity sound waves, as shown in Fig 17.14. Acoustic levitation provides a container-less environment for some experiments. Acoustic levitation occurs when sound waves interact and create a standing wave with nodes that can trap a particle.

Chladni Plates: A chladni plate consists of a flat metal sheet (usually circular or square) mounted on a central stalk attached to a strong base. When the plate oscillates in a particular mode of vibration, the nodes and antinodes that formed produce a complex but symmetrical patterns on its surface. The positions of these nodes and antinodes can be seen by sprinkling sand upon the plates. The sand will vibrate away from the antinodes and gather at the nodes.

Science Tidbit



Scan the above QR code then search the appeared link to see the video of an interesting experiment. It is a good demonstration to show how complicated modes of vibration can be formed on the chladni plates of different shapes and dimensions.

SUMMARY

- When a body moves to and fro about its mean position, then such motion is called vibratory or oscillatory motion.
- The complete round trip of an oscillating or vibrating body about its mean position is called oscillation or vibration.

- The distance of vibrating body from the equilibrium position on either side at any instant of time is known as displacement. Its SI unit is meter.
- The maximum displacement of a vibrating body from mean position is called amplitude. Its SI unit is meter.
- The time taken to complete one oscillation or vibration is called time period of the oscillation. It is denoted by T. Its SI unit is second.
- The number of vibrations or oscillations per unit time is called frequency.
- SHM is an oscillatory motion in which acceleration of a particle is directly proportional to its displacement from the mean position and is always directed towards the mean position.
- When a body moves in a circle, its projection undergoes simple harmonic motion on the diameter of the circle.
- The angle which specifies the displacement as well as the direction of a point executing SHM is called phase of the motion.
- Energy of a body executing SHM remains conserved. The energy oscillates between K.E and P.E, but their sum remains constant.
- A body is said to be executing free oscillation when it oscillates under the influence of restoring force without any external force acting on it.
- If the external forces make the object oscillating at the frequency of applied force rather than its natural frequency, then such oscillations are called forced oscillations.
- If the amplitude of oscillations decreases under damping forces, then such oscillations are called damped oscillations.
- The damping is said to be light when the amplitude of oscillations decreases gradually with time.
- When an oscillator comes to rest without any oscillation in the shortest time under a damping force, then such damping is called critical damping.
- Resonance is the phenomena in which the amplitude of vibration of an oscillator attains maximum value when deriving frequency equals the natural frequency of oscillator.
- A Rubens tube is used for demonstrating acoustic standing waves.
- Acoustic levitation is a method for suspending matter in air against gravity by using acoustic pressure from high intensity sound waves.

FORMULA SHEET

$f = \frac{1}{T}$ $\quad \omega = 2\pi f = \frac{2\pi}{T}$ $\quad F = -kx$

$\omega = \sqrt{\frac{k}{m}}$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$ $\quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$v = \omega\sqrt{x_o^2 - x^2}$ $\qquad a = -\omega^2 x$ $\qquad x = x_o \cos(\omega t + \varphi)$

$T = 2\pi\sqrt{\frac{l}{g}}$ $\qquad P.E = \frac{1}{2}k x^2$ $\qquad K.E = \frac{1}{2}mv^2 = \frac{1}{2}k(x_o^2 - x^2)$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following statements is true regarding the acceleration of the particle executing simple harmonic motion?
   A. Acceleration is zero at the mean position
   B. Acceleration is maximum at the mean position
   C. Acceleration is zero at the extreme position
   D. Acceleration is a constant
2) If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in seconds?
   A. 150 B. 2.5 C. 0.4 D. 0.0067
3) Which of the following relationship between the acceleration 'a' and the displacement 'x' of a particle involve SHM?
   A. a = 0.7 x B. $a = -200\,x^2$ C. a = -10 x D. $a = 100\,x^3$
4) A mass-spring system undergoes simple harmonic motion has amplitude A. When the kinetic energy of the object equals twice the potential energy stored in the spring, what is the position x of the object?
   A. A B. $\frac{A}{3}$ C. $\frac{A}{\sqrt{3}}$ D. 0
5) Suppose we replace the spring in a simple harmonic oscillator with a stronger spring, having twice the spring constant. What is the ratio of the new period of oscillation to the original period?
   A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$ C. 1 D. $\sqrt{2}$
6) A pinball machine uses a spring that is compressed 4.0 cm to launch a ball. If the spring constant is 13 N/m, what is the force on the ball at the moment the spring is released?
   A. 52 N B. 0.52 N C. 0.52 N m D. 5.2 N m
7) A particle is executing simple harmonic motion with period T. At time t = 0 it is at the equilibrium point. At which time is it furthest from the equilibrium point?
   A. 0.50 T B. 0.70 T C. 0.25 T D. 1.4 T
8) The acceleration of a body executing simple harmonic motion leads the velocity by what phase?
   A. 0 rad B π/8 rad C. π/4 rad D. π/2 rad

9) A simple pendulum has a period of 2.5 s. What is its period if its length is made four times larger?
A. 0.625 s B. 1.25 s C. 2.5 s D. 5 s

10) What is the frequency of a pendulum that swings at the rate of 45 cycles per minute?
A. 0.75 Hz B. 1.3 Hz C. 2700 Hz D. 60 Hz

11) A block at the end of a horizontal spring is pulled from equilibrium at x = 0 to x = A, and then released. Through what total distance does it travel in one full cycle of its motion?
A. $\frac{A}{2}$ B. A C. 2 A D. 4 A

12) A particle is vibrating simple harmonically with an acceleration of 16 cm s^{-2} when it is at a distance of 4 cm from the mean position. Its time period is:
A. 1 s B. 2.572 s C. 3.142 s D. 6.028 s

Short Questions

1) If we halve the length of a simple pendulum to its original length, what is the alteration in the period of this pendulum? What is its new frequency?
2) If the amplitude of vibration of a body executing SHM is doubled, what will happen to the maximum kinetic energy?
3) When marching soldiers are about to cross a bridge, they break steps. Why?
4) Suppose that a driving force has half frequency as compared to the frequency of an oscillator. Will it produce resonance? Similarly, if the driving frequency is twice the frequency of the oscillator, will it produce resonance?
5) Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you. Will you have to lengthen or shorten the pendulum to keep the correct time, other factors are remaining constant? Explain your answer.
6) Two mass-spring systems vibrate with simple harmonic motion. If the spring constants are equal and the mass of one system is twice that of the other, which system has a greater period?
7) Give some applications in which resonance plays an important role.
8) A simple pendulum is set into vibrations and left untouched, eventually stops, why?
9) Under what condition(s) the motion of a simple pendulum be simple harmonic motion?
10) At what position is the velocity of a particle executing simple harmonic motion a) maximum b) minimum?
11) Show that the motion of projection of a body revolving in a circle describes S.H.M.
12) sketch displacement-time graphs to illustrate light, critical and heavy damping.
13) Justify the importance of critical damping in a car suspension system.
14) Differentiate free and forced oscillations.
15) How the time period of a simple pendulum changes if mass of its bob is doubled? What change arises in time period of mass attached to a spring if same is done here?

Comprehensive Questions

1) *Define* and explain phase in simple harmonic motion.
2) Derive the expressions for instantaneous displacement, instantaneous velocity and acceleration of the projection of a particle moving in a circle.
3) Show that motion of a mass attached to a spring executes S.H.M.
4) Analyse the interchange between kinetic and potential energy during simple harmonic motion.
5) What is resonance? Give three of its applications in our daily life?
6) Justify some circumstances in which resonance should be avoided.
7) Derive equations for kinetic and potential energy of a body of mass m executing S.H.M.
8) Explain what is meant by damped oscillations?
9) Explain the relation between damping and sharpness of resonance.
10) Discuss the use of standing waves and resonance in (a) rubens tubes (b) chladni plates (c) acoustic levitation.
11) Analyze graphical representations of the variations of displacement, velocity and acceleration for simple harmonic motion.
12) Explain the terms light, critical and heavy damping, with the help of examples.

Numerical Problems

1) The amplitude of the motion of a mass attached to a spring is 2.48 m, while maximum speed of its mass is 4.36 m s^{-1}. What is the period of the motion? **(Ans: 3.57 s)**

2) A particle moves, whose displacement as a function of time is: x = 3.0 cos(2t), where distance is measured in meter and time in second.

a) Calculate the amplitude, the frequency, the angular frequency and the period of this particle?

b) Calculate the time at which the particle reaches the midpoint (i.e., x = 0) and the turning point?

(Ans: (a) 3.0 m, 0.318 Hz, 2.0 rad s^{-1}, 3.14 s (b) 0.785 s, 1.57 s)

3) A particle executes simple harmonic motion, that moves back and forth along x-axis between points −0.20 m and +0.20 m. The period of the motion is 1.2 s. At the time t = 0, the particle is at +0.20 m and its velocity is zero.

a) What is the frequency of the motion and the angular frequency?

b) What is the amplitude of the motion?

c) At what time will the particle reach the point x = 0? At what time will the particle reach the point x = 0.10 m?

d) What is the speed of the particle when it is at x = 0? What is the speed of the particle when it reaches the point x = 0.10 m?

(Ans: (a) 0.83 Hz, 5.2 rad s^{-1} (b) 0.20 m (c) 0.30 s, 0.20 s (d) 1.05 m s^{-1}, 0.91 m s^{-1})

4) A mass of 8.0 kg is attached to a spring and oscillates with amplitude of 0.25 m and has a frequency of 0.60 Hz. What is the energy of the motion? **(Ans: 3.6 J)**

5) Calculate the period and frequency of a 3.5 m long pendulum at the following locations:

a) at Karachi, where $g = 9.832 \text{ m s}^{-2}$.

b) at K-2, where $g = 9.782 \text{ m s}^{-2}$.

(Ans: (a) 3.747 s, 0.2669 Hz (b) 3.757 s, 0.2662 Hz)

6) In a car engine, a piston executes SHM with amplitude of 0.41 m. The engine is running at angular frequency of 2400 rpm (251 rad s^{-1}). What is the maximum speed of the piston?

(Ans: 103 m s^{-1})

7) A ball connected to a spring executes SHM. At $t = 0$, its displacement is 0.50 m and its acceleration is -0.72 m s^{-2}. The phase constant for its motion is 0.84 rad. What is the ball's displacement at $t = 3.4$ s? **(Ans: 0.15 m)**